Predicting and Measuring Knot Strength and Security

Introduction

Why do knots vary in strength and security? Why do these things depend on the type of rope and the situation in which the knot is used? Why are they so hard to model mathematically (in a way that provides reliable predictions)? What does that mean for the practical knot tier?

Knot strength

Knot strength or efficiency (resistance to breaking, relative to the same rope with no knot) depends on the extent to which force (from loading the rope) is transmitted to any place where the rope is weakened (usually by a tight arc in the rope, or perhaps a place at which the normally flexuous rope is made more rigid). The more force dissipated before tension falls on a weakening structure, the stronger the knot. Knots like the triple fisherman's (bend, not a misnamed anchor hitch) that dissipate considerable force by compression around the incoming rope before the first tight arc can be very strong. But the same apparent knot structure can be formed with different amounts of twisting of the rope and pre-tightening of the knot in ropes of different age and composition. Knots of the same apparent structure can also be subject to widely variable loading of their different parts, different directions of pull on the ropes entering the nub, and cyclic or shock loading. An eye knot used mid-rope in climbing provides a practical example. These factors can greatly influence knot efficiency, because they can influence structures that weaken the rope, the location of such structures, and the magnitude and timing of forces transmitted to such structures.

Emphasis is sometimes placed on the position within (or near) a knot at which breakage occurs. In some cases this can be studied by high-speed photography; but it is difficult to establish by inspection after breakage, because ropes commonly stretch under load and they are somewhat elastic. What commences at one point within the nub of a knot may lie elsewhere after breakage.

Knot security

Knot security (resistance to slippage, sometimes with a change of shape; and perhaps ultimately spillage) depends on friction. Friction in the knot depends on the coefficient of surface friction of the rope (and the underlying substrate in the case of hitches and toggled knots), the contact geometry (eg crossing number, contact area and location within the knot), and pressure at each point of contact. These factors vary with dressing and pre-tightening of the knot, twisting of the rope during tying, rope age, tension, and different directions of pull on the ropes entering the nub.

Aberrant loading (including cyclic loading) can greatly influence knot security. It is necessary to specify: "Security against what?". A steady increase in tension and slack shaking (for example) present very different challenges. A combination of challenges may be expected in some uses. An eye knot used as a mooring hitch for a boat subject to the incessant action of waves and tide provides a practical example.

Relationship in some modern ropes

Strength and security can be related, especially in modern ropes which are relatively slippery and susceptible to weakening by heat. In such ropes, slippage of a knot (even during tightening) may generate enough heat to weaken or break the loaded rope. Fishermen routinely lubricate their knots during tightening in modern lines, to minimize such effects. A 'pigtail' twist in fishing line near a knot after tightening is a sure sign of damage that has weakened the line in that region.

Effects of rope composition and size

Strength of cordage is highly dependent on the composition and the quality of materials used in construction, as well as the method used to keep parts together (soft-laid, hard-laid, braided and/or sheathed fibres). Different materials also vary in their sensitivity to tight curves, age, and environmental conditions such as water, pH, organic solvents and temperature.

Ropes of different compositions and constructions may show very different contact geometry in knots. Synthetic ropes vary widely, and exact composition is often unknown to users; so knot strength and security must be checked empirically in each unfamiliar rope.

Strength generally increases in proportion to area of cross-section (or the square of either diameter or circumference). But the factor applied to area generally decreases with rope size, because it is more difficult to ensure even loading of all fibres in larger ropes, As a few fibres break, a greater load falls on surrounding fibres and the break may propagate rapidly across a rope.

Even in the same rope type, not all factors scale in the same way with rope size. For example, surface area : volume ratio changes with size (because area increases as the square of diameter, whereas volume increases as the cube). The result is that, because knot strength and/or security depend on multiple surface effects and multiple effects within the rope, we usually lack the information needed to predict them accurately in different rope compositions and sizes.

Some modern rope fibres (eg HMPE) are very strong, but also very sensitive to tight curves and very low in surface friction. Ropes made from such fibres are unsuited to knotting, instead requiring long and gently tapered splices, or lashings with wide turns. However 'impregnated' HMPE twines are popular in knotted fishing nets (lower drag from thinner twine may outweigh the disadvantages from knots in netting).

Measurement

Ropes are inherently variable along their length and between batches.

Environmental factors differentially affect knots, and vary in time and space.

Subtle variations in dressing and pre-tightening can alter knot strength and security.

Therefore, practical measurement requires:

(a) careful documentation of the tested knot form and conditions; as well as

(b) sufficient replication and a randomized or patterned experimental design that allows valid statistical analysis for differences beyond those expected by chance.

The amount of replication needed can be predicted (statistically) from preliminary data, and it will increase with the desire to detect smaller differences between knots (treatments).

Such scientific testing (to provide a real contribution to knowledge rather that "data-free observations") is expensive and time consuming. No study published to date has a design needed for such confidence.

Modeling and experience

It is interesting that one of the oldest human technologies has proven so difficult to model. Some useful attempts to define some of the relevant factors mathematically can be found in the references below (and in works cited therein). One of the most recent studies (Patil *et al.* 2020) examined bending and stretching strain within (model) fibres, as well as topological functions derived from crossing and linking numbers, torque/ twist / writhe of intersecting strands, and friction in zones of intersection (circularity); but left material properties and contact geometry for future analysis. Useful progress has been made, but there is still a long way to go to develop a mathematical base from which to understand and predict practical knot behavior. As a simple example, I am not aware of any model which explains the greater variability in security of a granny relative to a reef knot; long noted by practical knot tiers.

The relative importance of factors affecting knot strength and security varies between rope sizes and compositions. Unfortunately, some of these factors can arise unexpectedly, some are difficult to measure in individual knot structures, and some are difficult to provide in advance (eg coefficient of surface friction in every substrate or rope composition and age of interest). This complicates mathematical modeling and prediction of knot strength and security. It puts greater reliance on the judgment of practical knot tiers about the prevailing conditions, and on the use of appropriate 'safety factors' and 'fail-safes' in critical applications.

To a lesser extent, the same may be said for other key features of knots such as ease of tying, ease of untying after a load (susceptibility to jamming), ease of confirmation by sight or feel, susceptibility to wear, knot dimensions under load, and even rope consumed in secure tying.

References

- Crowell B (2015) The physics of knots. http://www.lightandmatter.com/article/knots.html
- Johanns P, Grandgeorge P, Baek C, Sano TG, Maddocks JH & Reis PM (2021) The shapes of physical trefoil knots. *Extreme Mechanics Letters* 43, 101172. https://doi.org/10.1016/j.eml.2021.101172
- Krauel M (2005) The mathematical theory of hitches. https://www.math.ucla.edu/~radko/191.1.05w/matt.pdf
- Patil VP, Sandt JD, Kolle M & Dunkel J (2020) Topological mechanics of knots and tangles. *Science* 367, 71-75. <u>https://science.sciencemag.org/content/sci/367/6473/71.full.pdf</u>

Warner C (1996) Studies on the behaviour of knots. Chapter 10 in: Turner JC & van de Griend P (eds.) *History and Science of Knots*. World Scientific, Singapore. ISBN 9810224699. <u>https://www.watchmenofamerica.com/LEARNING-CENTER/Resource-</u> <u>Categories/MSC/MSC-002-PDF.pdf</u>