Rope Torsion and Knots

Torsion (torque, or twist + writhe when ends are constrained) of a rope seems to be regarded as mostly a nuisance by climbers whereas racing yachtsmen, dockers, mooring engineers and crane operators are much more careful about it. Torsion is vital to function in 'knotted' macromolecules.

In topological knot theory, torsion is evidently considered unimportant: a cross-over can be removed by Reidemeister move 1 (and thus ignored for the sake of simplicity in 'topological' mathematics). However, this maneuver adds twist into the closed line in which the 'theoretical knot' exists. In broader mathematical knot theory, writhe and twist are considered to be related 'geometric' qualities (see below). Some mathematicians still post papers that fail to recognize this difference. Papers that seem to include torsion are hard to understand for non-mathematicians.

This may help to explain why knot pairs such as *ABOK* #1406 (Whatnot) and *ABOK* #1407 (Grief knot) are commonly considered to be mere dressing variants of the same knot, whereas careful evaluation shows that they can only be interconverted in a closed line by introducing a cross-over (or rope torsion), typically outside the nub. Existing torsion in the ends will make this conversion harder or easier to achieve (depending on the direction of torsion). Of course, with short ends that are free to rotate, this cross-over or torsion is automatically relieved, and thus usually un-noticed. And in practice, the Whatnot is too easily converted into an insecure Grief knot to be considered 'safe' in rope. But the Whatnot structure is secure, and it is distinct from Grief knot structure. Evidently, 'geometric' qualities are very important in practical knotting. (In topological knot theory, both *ABOK* #1406 and #1407 reduce to the trefoil knot, so it does not help us much.)

Patil et al. (2020) noted that tightening a knot could introduce self-torque, and believed that opposite torques in adjacent crossing pairs caused locking between interacting lines and thus increased knot security.

Barnes (1951) noted reduced breaking strength if line torsion was added during knot tying.

Manufacturers of laid wire rope for cranes offer many insights into torsion effects. Susceptibility to heat from sliding under friction, and to other effects such as compression, varies between rope compositions. But many of the insights about torsion (including effects over pulley sheaves) are probably relevant in fibre rope, especially laid fibre rope.

In manufacture of laid fibre ropes, filaments are twisted into yarns, which are twisted into strands, which are in turn twisted into ropes. There may be several steps in each stage, depending on the finished rope size. Twisting converts each bundle into a more coherent, compact and flexible structure. The direction of twist is reversed at each stage to produce a more stable rope structure. Strength of the fibres is reduced by twisting, but this is necessary to obtain a stable laid rope. As load is applied to a laid rope, the rope tends to untwist. Modern laid ropes can be 'torque balanced' at a specific load range, but this will not eliminate all untwisting while the rope is loaded. Such twisting and untwisting complicates the estimation of torsion transmitted to a knot in a loaded rope.

There have been some useful studies of braided fibre rope, where twisting interferes with load-sharing between strands. A drop in break strength of around 4% to 7% per turn per metre is recorded in HMWPE tug lines.

But there has been little or no published consideration of effects of rope torsion outside the nub, on the strength of knotted rope. It might be especially important when rotation of the knot is constrained, as in most hitches.

Reference (in addition to links provided in the text)

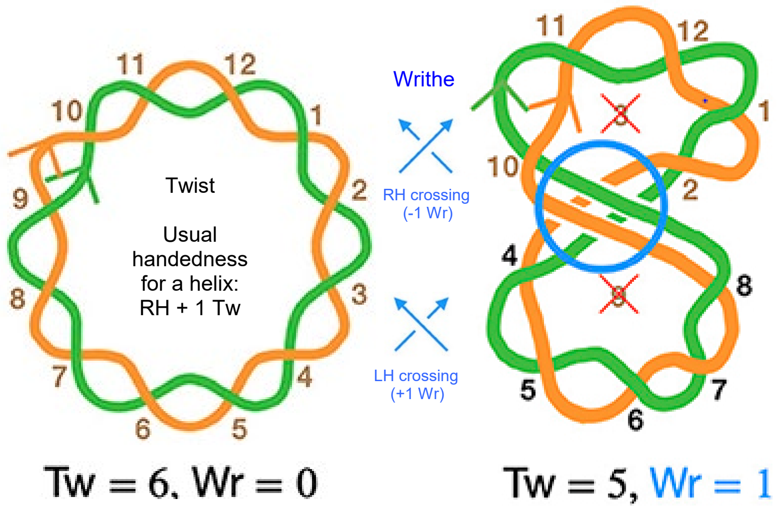
Barnes S (1951) *Anglers' Knots in Gut and Nylon.* Second Edition. Cornish Brothers Limited, Birmingham.

Mathematics of twist and writhe in knotted ropes

Consider two ropes wound over one another in a helix. Then topological definitions are:

- Linking number (Lk) is the number of times one rope passes over the other. It can be divided into two components:
    - Twist (Tw) is the number of helical turns (passing over of one specified rope within the helix).
    - Writhe (Wr) is the number of superhelical turns (passing over of both ropes when the entire helix crosses itself).

If the ends are joined in a 2-stranded structure such as a circle (as in plasmid DNA), or if the ropes are otherwise anchored, the total number of cross-overs is fixed. But twist and writhe can exchange for one another. Allowing for direction, Lk = Tw + Wr. Linking number is like torsion in that it reflects the total energy or strain captured in the final structure by ‘winding up’ the helix (beyond a relaxed state that varies depending on other properties of the ropes).



If two strands are wound over one another six times without writhe, then the ends are joined, Lk = Tw = 6 (left diagram).

If the ‘knot’ is allowed to exchange some of the twist for writhe (supercoiling), various shapes can arise. In the simple example shown, Lk = 5 + 1 = 6 (right diagram).

Modified from Junier et al. 2023, who discuss modelling in DNA. Some other publications get this wrong.

Things get confusing because (in mathematical knot theory) writhe is sometimes called “twist number”, and it is sometimes used as a synonym of Linking number (Lk). Although once considered to be a **topological knot** invariant, writhe (and twist) are today considered as geometric qualities in mathematical knot theory. But Lk is an invariant of **framed knots** and links (those with thickness of the components) which of course applies to all real knotting materials. Indeed, Reidmeister move 1 has been redefined to accommodate this reality.

A single rope can also be twisted, and some of the twists may be exchanged for writhes (if the rope bends and winds over itself in a helix). The shape of a particular rope under such torsion depends on the energies needed to bend and twist the rope. These mechanical properties vary between ropes in ways that are difficult to predict.

Anyone who is interested in how knots function should consider geometry and mechanics, not topology (which ignores most physical properties that are vital to knot function).